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# Numerical Analysis of Hydrodynamic Journal Bearing With Deterministic Surface Roughness

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**Abstract—***Hydrodynamic journal bearings have received the great attention of analytical and practical engineers in past few decades. Viscosity, Surface, and structure of liquid film are some important technical specification of Hydrodynamic lubrication. The viscosity is usually kept constant while performing the analysis with small loads for a Journal bearing with fluid lubrication. But modifications can be done in remaining two, for example, the structure of the fluid film and interacting surfaces. Modification of interacting surfaces of bearing means to change the surface texture of bearing. Surfaces undergo some type of roughness like waviness, directional roughness, lay etc due to the machining operation. It has great influence on pressure generated in the bearing as well as the load-carrying capacity of the bearing. In this paper, Finite Difference Method and MATLAB R2013 have been used for the numerical approach to see the effect of different types of surface roughness on pressure distribution in the journal bearing. Calculations are based on Reynolds equation for modified surface film.*

**Keywords***: Hydrodynamic lubrication, Surface texture, Finite difference method*

## 1. INTRODUCTION

For over a decade the hydrodynamic theory of rough surfaces has been studied with considerable interest. Hydrodynamic bearings which support high-speed rotating machinery are in use for a long term, generally over 10 years, so the bearing and journal surfaces may become roughened by reasons such as wear, impulsive damage, foreign particles, cavitation erosion, rust, etc. The purpose of this study is to see the influence of the roughness parameter on bearing characteristics. To probe the question how surface irregularities should be mathematically described, the paper considers the surface profiles as purely sinusoidal or triangular. In the numerical analysis Reynolds equation has been modified by considering the film thickness as sinusoidal or triangular wave function of θ (theta).[7-11].

## 2. LITERATURE REVIEW

**Christensen et al. (1973)** [1] described the application of this theory to the analysis of the full journal bearing of finite width. The analysis demonstrates how the roughness influences the characteristics of the bearing and also shows how roughness interacts with features of nominal geometry and operating factors to determine the bearing response.

**Patir et al. (1978)** [2] applied new approach is utilized to determine the effects of surface roughness on partially lubricated contacts. An average Reynolds equation for rough surfaces is defined in terms of pressure and shear flow factors, which are obtained by numerical flow simulation. Through the use of measured or numerically generated rough surfaces, any three dimensional roughness structure can be analyzed with this method.

**Tripp et al. (1983)** [3] in their present study the average flow model of Patir and Cheng for obtaining an average Reynolds equation in the presence of two dimensional surface roughness is extended and generalized. Terms in the series are evaluated using the unperturbed Green function, which permits ensemble averaging to be performed directly on the solution. Calculations are carried to second order, which involves only two point correlation functions of the two rough surfaces. The theory displays the dependence of the flow factors on the roughness parameters in simple closed form, leading to improved understanding of the average flow method.

**Bayada et al. (1988)** [4] described recent advances in the mathematical analysis of problems described by several small parameters equations are used to revisit the general roughness problem. We are led to distinguish three different cases in which the periodic roughness wavelength is on the order of, greater or shorter than the mean thickness of the gap.

**Burstein et al. (2007)** [5] studied a spatial one-dimensional time-dependent mathematical model of roughened sliding surfaces with a sinusoidal profile was developed and applied in the context of their fluid lubrication. The opposing surfaces have identical average roughness and wavelength characteristics. The hydrodynamic pressure distribution equation is derived for different peak-hollow numbers and roughness depths. Matlab (the Mathworks, Inc., Natick, MA, USA) calculations show a significant effect of roughness with a small number of peak-hollow waves on the above parameter.

**Wada et al. (1971)** [6] in their paper, they studied the applications of the finite-element method for hydrodynamic lubrication of infinite-width bearings are presented. It is claimed that the finite-element method is able to obtain the accurate results by using a few elements.

## 3. NUMERICAL ANALYSIS

### 3.1 Governing Equations

The pressure profile in finite journal bearing is governed by following set of equations:

**Reynolds Equation**: Assuming laminar, incompressible, inertia less, Newtonian fluid; the Reynolds equation in Cartesian coordinate system for a finite journal bearing obtained from momentum equation is given by:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial y}\right)-6u_s\frac{\partial(\rho h)}{\partial x}=0$$

Where, $x = R\theta$

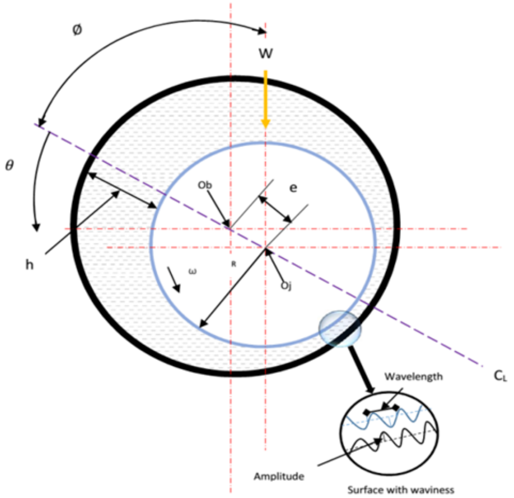


**Fig. 1: Schematic diagram of hydrodynamic journal bearing.**

**Film Thickness Equation:**

The film thickness equation for a finite journal bearing is given by:

$$h = c(1+\varepsilon\cos\theta)$$

With the introduction of surface waviness on the surface, the film thickness equation takes the form:

$$h = c(1+\varepsilon\cos\theta) + a\sin\left(\frac{2\pi L}{\lambda}\right)$$

With the introduction of Triangular surface roughness on the surface of bearing, the film thickness equation takes the form:

$$h = c(1+\varepsilon\cos\theta) + a.ArcSin\left(Sin\left(\frac{2\pi L}{\lambda}\right)\right)$$

**Non Dimensional Equations**

In addition to reducing the number of parameters, non-dimensionalized equation helps to gain a greater insight into the relative size of various terms present in the equation. Following appropriate selecting of scales for the Non-dimensionalization process, this leads to an identification of small terms in the equation. Neglecting the smaller terms against the bigger ones allows for the simplification of the situation. The non-dimensional form of an equation is given below.

**Film thickness equation:** By substituting

$$H = \frac{h}{C} and\ A = \frac{a}{C}$$

Film thickness is made dimensionless to take the form

$$H = (1+\varepsilon\cos\theta) + A\sin(\theta * n)$$

For triangular surface roughness, the film thickness equation becomes,

$$h = c(1+\varepsilon\cos\theta) + a.ArcSin(\,Sin(\theta * n))$$

Where n is the no. of waves on the surface.

In order to form dimensionless Reynolds equation following substitutions are done in Reynolds equation. $x = R\theta$, $Y = {}^{y}/_{L}$ and $P = \frac{p*C^2}{6RU\eta_0}$.

The non-dimensional form of Reynolds equation after above substitution takes the form

$$\frac{1}{\bar{\eta}}\left[3(1+\varepsilon\cos\theta)^2(-\varepsilon\sin\theta)\frac{\partial P}{\partial\theta}+(1+\varepsilon\cos\theta)^3\frac{\partial}{\partial\theta}\left(\frac{\partial P}{\partial\theta}\right)\right]+\frac{H^3}{\bar{\eta}}\left(\frac{R}{L}\right)^2\left[\frac{\partial}{\partial L}\left(\frac{\partial P}{\partial L}\right)\right]=-\varepsilon\sin\theta$$

### 3.2 Discretization

In this section, discrete equations will be built up from the first principle. This process requires that a set of sample points inside the domain are chosen at which the equations will be satisfied. The more the points will be chosen, the closer the point will be to each other and hence the more accurate the solution. However as the number of mesh point increases, so does the amount of computational work required to solve the problem.

In order to discretize the Pressure equation, the Finite Difference Method is used. By Taylor series we have

$$f(x+h,y) = f(x,y) + \frac{\partial f}{\partial x}h + \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\frac{h^2}{2!} + \cdots$$

Similarly

$$f(x-h,y) = f(x,y) - \frac{\partial f}{\partial x}h + \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\frac{h^2}{2!} + \cdots$$

Subtracting above equations, we get

$$\frac{\partial f}{\partial x} = \frac{f(x+h,y) - f(x-h,y)}{2h} + o(h^2)$$

For any node i,j we have

$$\left(\frac{\partial f}{\partial x}\right)_{i,j} = \frac{f_{i+1,j} - f_{i-1,j}}{2h} + o(h^2)$$

And

$$\left(\frac{\partial f}{\partial y}\right)_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2k} + o(k^2)$$

Where, $x_{i+1} - x_i = h$ and $y_{j+1} - y_j = k$

Similarly

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{f(x+h,y) + f(x-h,y) - 2f(x,y)}{h^2} + o(h^2)$$

For any node i,j we have

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)_{i,j} = \frac{f_{i+1,j} + f_{i-1,j} - 2f_{i,j}}{h^2} + o(h^2)$$

And

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)_{i,j} = \frac{f_{i,j+1} + f_{i,j-1} - 2f_{i,j}}{k^2} + o(k^2)$$

**3.4.1 Reynolds equation:** In order to calculate the Non-dimensional pressure, it is first necessary to discretize the dimensionless pressure. The discrete form of pressure equation is given as,

Considering sinusoidal roughness on bearing surface,

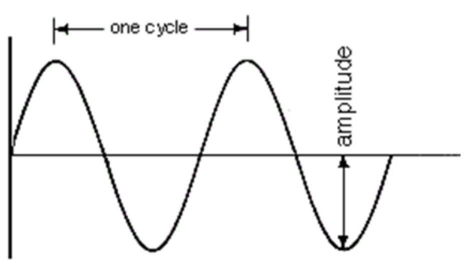


**Fig. 2: Representation of waviness**

Then Non-dimensionalized Reynolds equation takes the form,

$$\frac{1}{\bar{\eta}}\left[[3(H)^2(-\varepsilon\,sin\theta)]\left\{\frac{P_{i+1,j} - P_{i-1,j}}{2\Delta\theta}\right\} + [(H)^3]\left\{\frac{P_{i+1,j} + P_{i-1,j} - 2P_{i,j}}{\Delta\theta^2}\right\}\right] + \frac{H^3}{\bar{\eta}}\left(\frac{R}{L}\right)^2\left[\frac{P_{i,j+1} + P_{i,j-1} - 2P_{i,j}}{\Delta l^2}\right] = -\varepsilon\,sin\theta$$

Where, $\Delta\theta = \frac{\pi}{180}\, and\ \Delta l = L/180$

Now by rearranging the above equation we get Non-Dimensionless pressure equation for sinusoidal roughness,

$$P_{i,j} = P_{i+1,j}\left(-\frac{A_2}{A_1}\right) + P_{i-1,j}\left(-\frac{A_3}{A_1}\right) + P_{i,j+1}\left(-\frac{A_4}{A_1}\right) + P_{i,j-1}\left(-\frac{A_5}{A_1}\right) + \frac{A_6}{A_1}$$

Where,

$$A_1 = \frac{-2(H)^3}{\Delta\theta^2\bar{\eta}} - \frac{2H^3}{\Delta l^2\bar{\eta}}\left(\frac{R}{L}\right)^2$$

$$A_2 = \frac{(H)^3}{\Delta\theta^2\bar{\eta}} - \frac{3\varepsilon\,sin\theta(H)^2}{\bar{\eta}\,2\Delta\theta} + \frac{3A.n.(H)^2.Cos\,(n\theta)}{\bar{\eta}\,2\Delta\theta}$$

$$A_3 = \frac{(H)^3}{\Delta\theta^2\bar{\eta}} + \frac{3\varepsilon\,sin\theta(H)^2}{\bar{\eta}\,2\Delta\theta} - \frac{3A.n.(H)^2.Cos\,(n\theta)}{\bar{\eta}\,2\Delta\theta}$$

$$A_4 = A_5 = \frac{H^3}{\Delta l^2\bar{\eta}}\left(\frac{R}{L}\right)^2$$

$$A_6 = -\,\varepsilon\,sin\theta + A.n.Cos(n\theta)$$

Considering triangular roughness on bearing surface

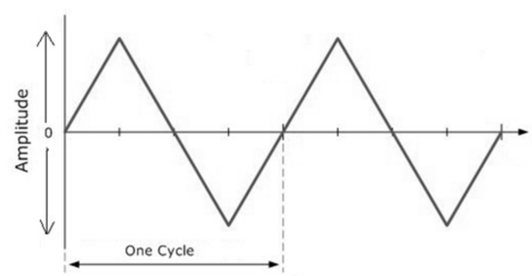


**Fig. 3: Representation of triangular roughness**

$$\frac{1}{\bar{\eta}}\left[\left[3(H)^2(-\varepsilon\, sin\theta) + \frac{2An}{\pi}\right]\left\{\frac{P_{i+1,j} - P_{i-1,j}}{2\Delta\theta}\right\} + [(H)^3]\left\{\frac{P_{i+1,j} + P_{i-1,j} - 2P_{i,j}}{\Delta\theta^2}\right\}\right] + \frac{H^3}{\bar{\eta}}\left(\frac{R}{L}\right)^2\left[\frac{P_{i,j+1} + P_{i,j-1} - 2P_{i,j}}{\Delta l^2}\right] = -\varepsilon\, sin\theta + \frac{2An}{\pi}$$

Where, $\Delta\theta = \frac{\pi}{180}\, and\ \Delta l = L/180$

Now by rearranging the above equation we get Non-Dimensionless pressure equation for triangular roughness,

$$P_{i,j} = P_{i+1,j}\left(-\frac{A_2}{A_1}\right) + P_{i-1,j}\left(-\frac{A_3}{A_1}\right) + P_{i,j+1}\left(-\frac{A_4}{A_1}\right) + P_{i,j-1}\left(-\frac{A_5}{A_1}\right) + \frac{A_6}{A_1}$$

Where,

$$A_1 = \frac{-2(H)^3}{\Delta\theta^2\bar{\eta}} - \frac{2H^3}{\Delta l^2\bar{\eta}}\left(\frac{R}{L}\right)^2$$

$$A_2 = \frac{(H)^3}{\Delta\theta^2\bar{\eta}} - \frac{3\varepsilon\, sin\theta(H)^2}{\bar{\eta}\, 2\Delta\theta} + \frac{3A.n.(H)^2}{\bar{\eta}\,\Delta\theta\,\pi}$$

$$A_3 = \frac{(H)^3}{\Delta\theta^2\bar{\eta}} + \frac{3\varepsilon\, sin\theta(H)^2}{\bar{\eta}\, 2\Delta\theta} - \frac{3A.n.(H)^2}{\bar{\eta}\,\Delta\theta\,\pi}$$

$$A_4 = A_5 = \frac{H^3}{\Delta l^2\bar{\eta}}\left(\frac{R}{L}\right)^2$$

$$A_6 = -\,\varepsilon\, sin\theta + \frac{2An}{\pi}$$

## 4. NUMERICAL METHOD

The iterative procedure begins with an initial pressure field prescribed to determine the integrals in the generalized Reynolds equation. Reynolds equation is then solved for pressure distribution in the bearing. Reynolds boundary condition is satisfied by Christopher-son algorithm, i.e., by setting negative pressure equal to zero as and when it appears in the process of iteration. The process is repeated with the new pressure distribution by solving the Reynolds equation.

In the present analysis, the angle in X-direction and length in Y-direction has been divided into 361 nodes which is equal to 360 equal intervals respectively. The initial non-dimensional pressure at each node is assigned zero value. The iteration process is continued till the convergence criteria are satisfied, i.e., the difference between values at a node in two successive iterations is very small. Generally after 30-40 iterations the value converges, as shown in the result. The convergence criteria are:

For Non-dimensional Pressure

$$\frac{\left|\left(\sum P_{i,j}\right)_{N-1} - \left(\sum P_{i,j}\right)_N\right|}{\left|\left(\sum P_{i,j}\right)_N\right|} \leq 0.00001$$

As the following convergence criteria are satisfied after certain iterations, the iteration is stopped to calculate pressure at each node. The maximum pressure is also obtained..

## 5. RESULTS

In this work, using MATLAB software package results have been obtained in the form of graphs.

**Numerical Solution with MATLAB (R2013a)**

In this section various operating conditions for finite journal bearing are chosen to generate pressure distribution.

**Bearing without surface roughness**

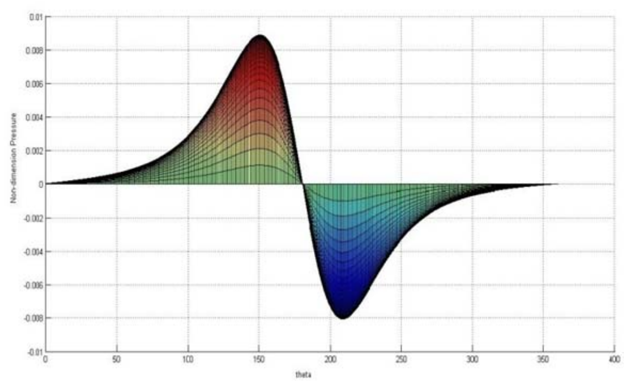

**Fig. 4 : Non dimensional pressure distribution without surface roughness**

**Bearing with sinusoidal roughness**

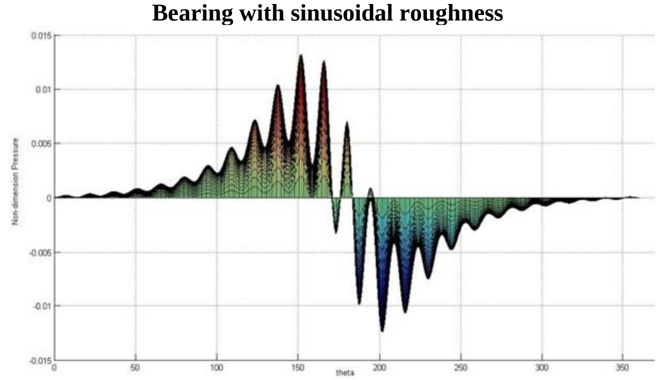

**Fig. 5: Non dimensional pressure distribution with sinusoidal roughness**

**Bearing with Triangular surface Roughness**

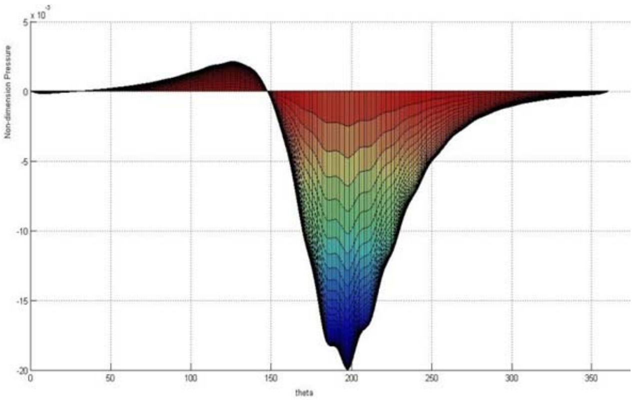


**Fig.6 : Non dimensional pressure distribution with Triangular Roughness**

## 6. CONCLUSION

From the results it has been found that the sinusoidal and triangular roughness in hydrodynamic journal bearing causes large fluctuations of pressure. The maximum and the minimum non dimensional pressure are higher than the smooth journal bearing. But the average pressure over the bearing is lower than the smooth bearing. Fluctuations produced in the pressure will cause high frequency vibrations in the machinery. If the roughness cycles increases with its amplitude (2% of clearance) affects the hydrodynamic action in the bearing and it leads bearing to failure.

## REFERENCES:


[1] Christensen, H., K. Tonder. "The hydrodynamic lubrication of rough journal bearings." Journal of Lubrication Technology 95.2 (1973): 166-172.

[2] Patir, Nadir, H. S. Cheng. "An average flow model for determining effects of three-dimensional roughness on partial hydrodynamic lubrication." Journal of lubrication Technology 100.1 (1978): 12-17.

[3] Tripp, J. H. "Surface roughness effects in hydrodynamic lubrication: the flow factor method." Journal of lubrication technology 105.3 (1983): 458-463.

[4] Bayada, G., M. Chambat. "New models in the theory of the hydrodynamic lubrication of rough surfaces." Journal of tribology 110.3 (1988): 402-407.

[5] Burstein, L. "Two-sided surface roughness and hydrodynamic pressure distribution in lubricating films." Lubrication Science 19.2 (2007): 101-112.

[6] Wada.S, H.Hayashi, M.Migita. "Application Bulletin of the JSME of Finite-Element Method to Hydrodynamic Lubrication Problems: Part 1, Infinite-Width Bearings." Bulletin of JSME 14.77 (1971): 1222-1233.

[7] Khonsari MM. (1987) "A review of thermal effects in hydrodynamic bearings, part II: journal bearing". ASLE Trans, 30(1), pp: 26–33.

[8] Cameron A, Wood WL. (1949) "The full journal bearing". Proceedings of the Institution of Mechanical Engineers,161, pp:59–64

[9] Reynolds O. (1886)"On the theory of lubrication and is application to Beauchamp, experiments". Phil Trans Proc Roy Soc.

[10] Yu, H., Wang, X. and Zhou, F., 2010. Geometric shape effects of surface texture on the generation of hydrodynamic pressure between conformal contacting surfaces. Tribology Letters, 37(2), pp.123-130.

[11] Christensen, H. and Tonder, K., 1971. The hydrodynamic lubrication of rough bearing surfaces of finite width. Journal of Lubrication Technology, 93(3), pp.324-329.